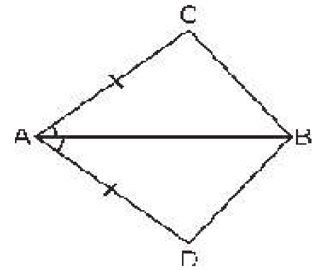
7 TRIANGLES

EXERCISE 7.1

Q.1. *In quadrilateral ACBD, AC = AD and AB bisects ∠A (see Fig.). Show that ΔABC ≅ ΔABD. What can you say about BC and BD?*



Sol. In ΔABC and ΔABD, we have

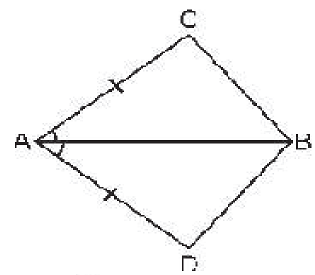
AC = AD [Given]

∠CAB = ∠DAB [∵ AB bisects ∠A]

AB = AB [Common]

∴ ΔABC ≅ ΔABD. [By SAS congruence] **Proved.**

Therefore, BC = BD. (CPCT). **Ans.**

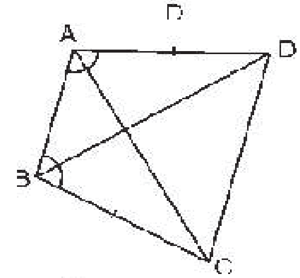


Q.2. *ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA (see Fig.). Prove that*

(i) Δ ABD ≅ Δ BAC

(ii) BD = AC

(iii) ∠ ABD = ∠ BAC



Sol. In the given figure, ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA.

In ΔABD and ΔBAC, we have

AD = BC [Given]

∠DAB = ∠CBA [Given]

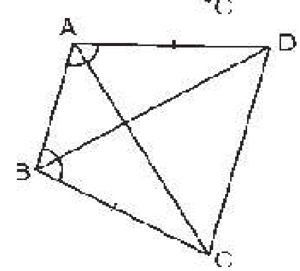
AB = AB [Common]

∴ ΔABD ≅ ΔBAC [By SAS congruence]

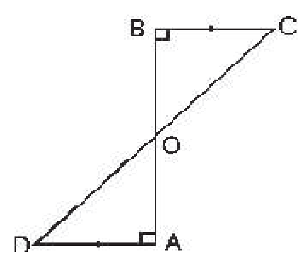
∴ BD = AC [CPCT]

and ∠ABD = ∠BAC [CPCT]

Proved



Q.3. *AD and BC are equal perpendiculars to a line segment AB (see Fig.). Show that CD bisects AB.*



Sol. In ΔAOD and ΔBOC, we have,

∠AOD = ∠BOC [Vertically opposite angles)

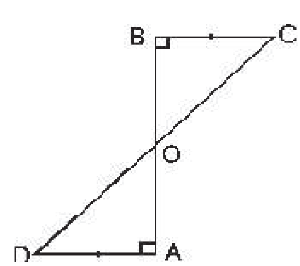
∠CBO = ∠DAO [Each = 90°]

and AD = BC [Given]

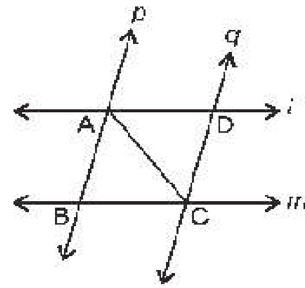
∴ ΔAOD ≅ ΔBOC [By AAS congruence]

Also, AO = BO [CPCT]

Hence, CD bisects AB **Proved.**



Q.4. *l and m are two parallel lines intersected by another pair of parallel lines p and q (see Fig.). Show that* $\Delta ABC \cong \Delta CDA$.



Sol. In the given figure, ABCD is a parallelogram in which AC is a diagonal, i.e., AB || DC and BC || AD.

In Δ ABC and Δ CDA, we have,

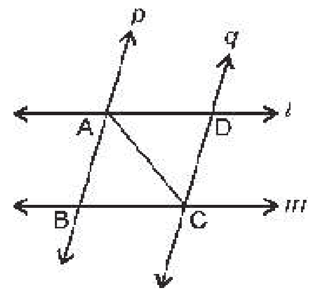
$\angle BAC = \angle DCA$ [Alternate angles]

$\angle BCA = \angle DAC$ [Alternate angles]

$AC = AC$ [Common]

$\therefore$ $\Delta ABC \cong \Delta CDA$ [By ASA congruence]

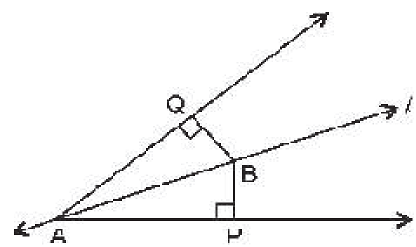
Proved.



Q.5. *Line l is the bisector of an angle A and B is any point on l. BP and BQ are perpendiculars from B to the arms of* $\angle A$ *(see Fig.). Show that :*

(i) $\Delta APB \cong \Delta AQB$

(ii) BP = BQ or B is equidistant from the arms of $\angle A$.



Sol. In Δ APB and Δ AQB, we have

$\angle PAB = \angle QAB$ [l is the bisector of $\angle A$]

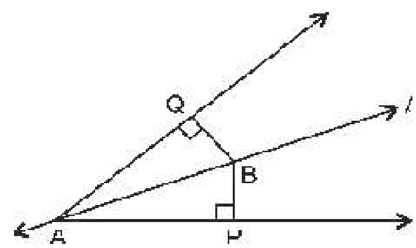
$\angle APB = \angle AQB$ [Each = 90°]

$AB = AB$ [Common]

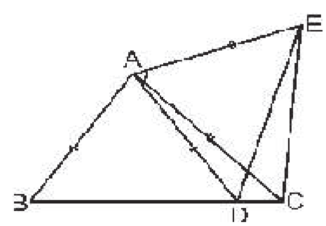
$\therefore$ $\Delta APB \cong \Delta AQB$ [By AAS congruence]

Also, BP = BQ [By CPCT]

i.e., B is equidistant from the arms of $\angle A$. **Proved**



Q.6. *In the figure, AC = AE, AB = AD and* $\angle BAD = \angle EAC$. *Show that BC = DE.*



Sol. $\angle BAD = \angle EAC$ [Given]

$\Rightarrow$ $\angle BAD + \angle DAC = \angle EAC + \angle DAC$ [Adding $\angle DAC$ to both sides]

$\Rightarrow$ $\angle BAC = \angle EAC$ ... (i)

Now, in ΔABC and ΔADE, we have

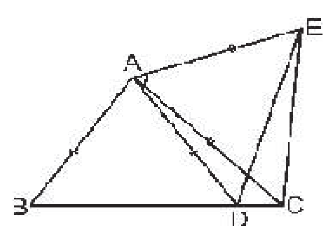
$AB = AD$ [Given]

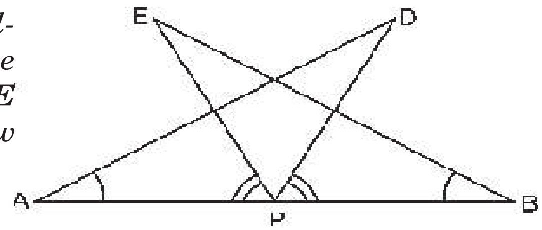
$AC = AE$ [Given)

$\Rightarrow$ $\angle BAC = \angle DAE$ [From (i)]

$\therefore$ $\Delta ABC \cong \Delta ADE$ [By SAS congruence]

$\Rightarrow$ $BC = DE$. [CPCT] **Proved.**





Q.7. *AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB (see Fig.). Show that*

(i) $\Delta DAP \cong \Delta EBP$ (ii) $AD = BE$

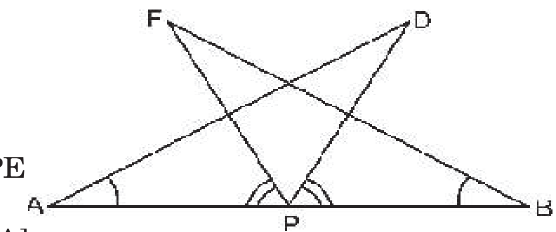
Sol. In ΔDAP and ΔEBP, we have

AP = BP [Q P is the mid-point of line segment AB]

∠BAD = ∠ABE [Given]

∠EPB = ∠DPA

[Q ∠EPA = ∠DPB ⇒ ∠EPA + ∠DPE = ∠DPB + ∠DPE]



∴ Δ DPA ≅ Δ EPB [ASA]

⇒ AD = BE [By CPCT] **Proved.**

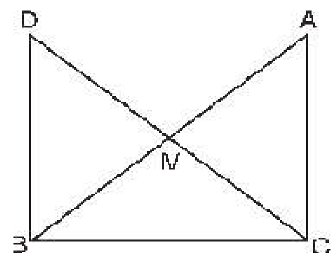
Q.8. *In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig.). Show that :*

(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ *is a right angle.*

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2}AB$



Sol. In ΔBMB and ΔDMC, we have

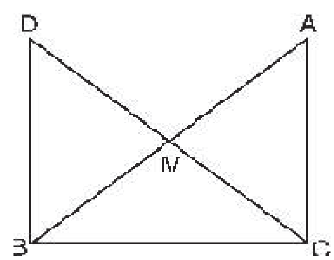
(i) DM = CM [Given]

BM = AM [Q M is the mid-point of AB]

∠DMB = ∠AMC [Vertically opposite angles]

∴ ΔAMC ≅ ΔBMD [By SAS]

Proved.



(ii) AC || BD [Q ∠DBM and ∠CAM are alternate angles]

⇒ ∠DBC + ∠ACB = 180° [Sum of co-interior angles]

[Q ∠ACB = 90°] **Proved.**

⇒ ∠DBC = 90° **Proved.**

(iii) In ΔDBC and ΔACB, we have

DB = AC [CPCT]

BC = BC [Common]

∠DBC = ∠ACB [Each = 90°]

∴ ΔDBC ≅ ΔACB [By SAS] **Proved.**

(iv) ∴ AB = CD [CPCT]

⇒ $\frac{1}{2}AB = \frac{1}{2}CD$

Hence, $\frac{1}{2}AB = CM$ [$CM = \frac{1}{2}CD$] **Proved.**

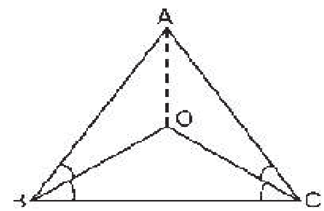
7 | TRIANGLES

EXERCISE 7.2

Q.1. *In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that :*
(i) OB = OC (ii) AO bisects ∠A.

Sol. (i) AB = AC ⇒ ∠ABC = ∠ACB [Angles opposite to equal sides are equal]

$\frac{1}{2}$∠ABC = $\frac{1}{2}$∠ACB



⇒ ∠CBO = ∠BCO [OB and OC are bisectors of ∠B and ∠C respectively]

⇒ OB = OC [Sides opposite to equal angles are equal]

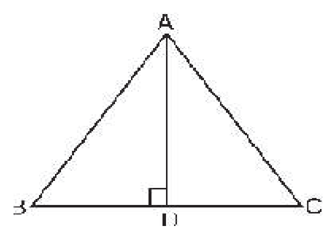
Again, ∠$\frac{1}{2}$ABC = $\frac{1}{2}$∠ACB

⇒∠ABO = ∠ACO [∴ OB and OC are bisectors of ∠B and ∠C respectively]

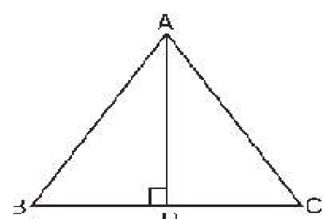
In ΔABO and ΔACO, we have

AB = AC [Given]
OB = OC [Proved above]
∠ABO = ∠ACO [Proved above]
∴ ΔABO ≅ ΔACO [SAS congruence]
⇒ ΔABO = ∠CAO [CPCT]
⇒ AO bisects ∠A **Proved.**

Q.2. *In Δ ABC, AD is the perpendicular bisector of BC (see Fig.). Show that Δ ABC is an isosceles triangle in which AB = AC.*



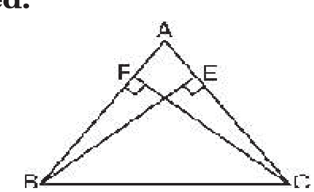
Sol. In ΔABD and ΔACD, we have



∠ADB = ∠ADC [Each = 90°]
BD = CD [∵ AD bisects BC]
AD = AD [Common]
∴ ΔABD ≅ ΔACD [SAS]
∴ AB = AC [CPCT]

Hence, ΔABC is an isosceles triangle. **Proved.**

Q.3. *ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig.). Show that these altitudes are equal.*

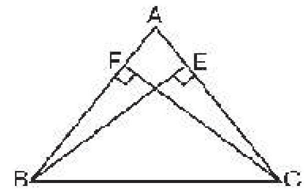


Sol. In ΔABC,

AB = AC [Given]

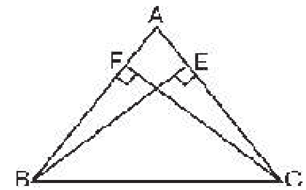
⇒ ∠B = ∠C [Angles opposite to equal sides of a triangle are equal]

Now, in right triangles BFC and CEB,

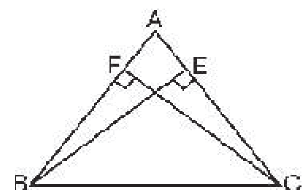


$\angle BFC = \angle CEB$ [Each = 90°]
$\angle FBC = \angle ECB$ [Pproved above]
$BC = BC$ [Common]
$\therefore$ $\Delta BFC \cong \Delta CEB$ [AAS]
Hence, $BE = CF$ [CPCT] **Proved.**

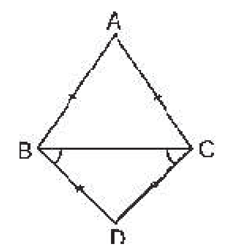
Q.4. *ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Fig.). Show that*
(i) $\Delta ABE \cong \Delta ACF$
(ii) $AB = AC$, *i.e., ABC is an isosceles triangle.*



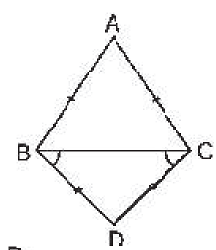
Sol. (i) In Δ ABE and ACF, we have
$BE = CF$ [Given]
$\angle BAE = \angle CAF$ [Common]
$\angle BEA = \angle CFA$ [Each = 90°]
So, $\Delta ABE \cong \angle ACF$ [AAS] **Proved.**
(ii) Also, $AB = AC$ [CPCT]
i.e., ABC is an isosceles triangle **Proved.**



Q.5. *ABC and DBC are two isosceles triangles on the same base BC (see Fig.). Show that* $\angle ABD = \angle ACD$.

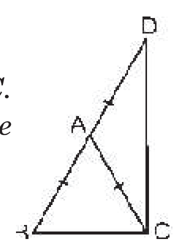


Sol. In isosceles ΔABC, we have
$AB = AC$
$\angle ABC = \angle ACB$ ...(i)
[Angles opposite to equal sides are equal]
Now, in isosceles ΔDCB, we have
$BD = CD$
$\angle DBC = \angle DCB$ ...(ii)
[Angles opposite to equal sides are equal]

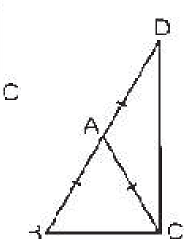


Adding (i) and (ii), we have
$\angle ABC + \angle DBC = \angle ACB + \angle DCB$
$\Rightarrow$ $\angle ABD = \angle ACD$ **Proved.**

Q.6. Δ *ABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see Fig.). Show that* $\angle BCD$ *is a right angle.*



Sol. $AB = AC$ [Given]
$\angle ACB = \angle ABC$ ...(i)
[Angles opposite to equal sides are equal]



$AB = AD$ [Given]
$\therefore$ $AD = AC$ [$\because$ AB = AC]
$\therefore$ $\angle ACD = \angle ADC$ ...(ii) [Angles opposite to equal sides are equal]
Adding (i) and (ii)
$\angle ACB + \angle ACD = \angle ABC + \angle ADC$
$\Rightarrow$ $\angle BCD = \angle ABC + \angle ADC$ ...(iii)
Now, in ΔBCD, we have
$\angle BCD + \angle DBC + \angle BDC = 180°$ [Angle sum property of a triangle]

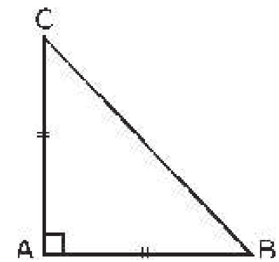
$\therefore \quad \angle BCD + \angle BCD = 180°$

$\Rightarrow \quad 2\angle BCD = 180°$

$\Rightarrow \quad \angle BCD = 90°$

Hence, $\angle BCD = 90°$ or a right angle **Proved.**

Q.7. *ABC is a right angled triangle in which $\angle A = 90°$ and AB = AC. Find $\angle B$ and $\angle C$.*



Sol. In ΔABC, we have

$\angle A = 90°$
and AB = AC } [Given]

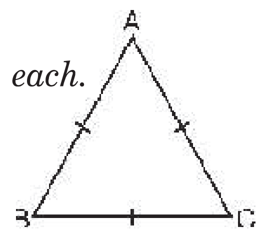
We know that angles opposite to equal sides of an isosceles triangle are equal.

So, $\angle B = \angle C$

Since, $\angle A = 90°$, therefore sum of remaining two angles = 90°.

Hence, $\angle$**B** = $\angle$**C = 45°** **Answer.**

Q.8. *Show that the angles of an equilateral triangle are* 60° *each.*



Sol. As ΔABC is an equilateral.

So, AB = BC = AC

Now, AB = AC

$\Rightarrow \angle ACB = \angle ABC$ ...(i)

[Angles opposite to equal sides of a triangle are equal]

Again, BC = AC

$\Rightarrow \angle BAC = \angle ABC$ ...(ii) [same reason]

Now, in ΔABC,

$\angle ABC + \angle ACB + \angle BAC = 180°$ [Angle sum property of a triangle]

$\Rightarrow \angle ABC + \angle ABC + \angle ABC = 180°$ [From (i) and (ii)]

$\Rightarrow 3\angle ABC = 180°$

$$\Rightarrow \angle ABC = \frac{180°}{3} = 60°$$

Also, from (i) and (ii)

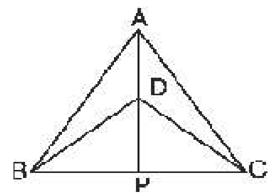
$\angle ACB = 60°$ and $\angle BAC = 60°$

Hence, each angle of an equilateral triangle is 60° **Proved.**

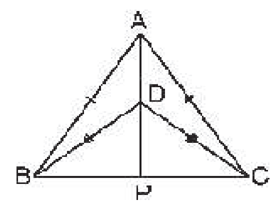
7 TRIANGLES

EXERCISE 7.3

Q.1. *ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig.). If AD is extended to intersect BC at P, show that*



(i) $\Delta ABD \cong \Delta ACD$
(ii) $\Delta ABP \cong \Delta ACP$
(iii) AP bisects $\angle A$ as well as $\angle D$.
(iv) AP is the perpendicular bisector of BC.



Sol. (i) In ΔABD and ΔACD, we have

AB = AC [Given]
BD = CD [Given]
AD = AD [Common]
$\therefore$ $\Delta ABD \cong \Delta ACD$ [SSS congruence] **Proved.**

(ii) In ΔABP and ΔACP, we have

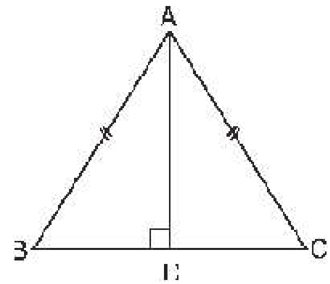
AB = AC [Given]
$\angle BAP = \angle CAP$ [Q $\angle BAD = \angle CAD$, by CPCT]
AP = AP [Common]
$\therefore$ $\Delta ABP \cong \Delta ACP$ [SAS congruence] **Proved.**

(iii) $\Delta ABD \cong \Delta ADC$ [From part (i)]
$\Rightarrow$ $\angle ADB = \angle ADC$ (CPCT)
$\Rightarrow$ $180° - \angle ADB = 180° - \angle ADC$
$\Rightarrow$ Also, from part (ii), $\angle BAPD = \angle CAP$ [CPCT]
$\therefore$ AP bisects ÐA as well as $\angle D$. **Proved.**

(iv) Now, BP = CP
and $\angle BPA = \angle CPA$ [By CPCT]
But $\angle BPA + \angle CPA = 180°$ [Linear pair]
So, $2\angle BPA = 180°$
Or, $\angle BPA = 90°$
Since BP = CP, therefore AP is perpendicular bisector of BC.
Proved.

Q.2. *AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that*

(i) AD bisects BC *(ii) AD bisects ∠A.*



Sol. (i) In ΔABD and ΔACD, we have

∠ADB = ∠ADC [Each = 90°]

AB = AC [Given]

AD = AD [Common]

∴ ΔABD ≅ ΔACD [RHS congruence]

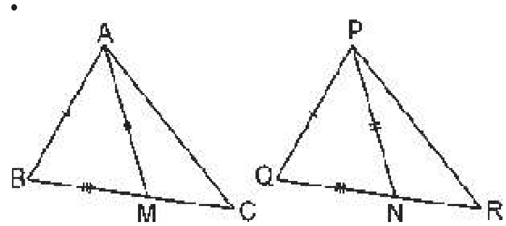
∴ BD = CD [CPCT]

Hence, AD bisects BC.

(ii) Also, ∠BAD = ∠CAD

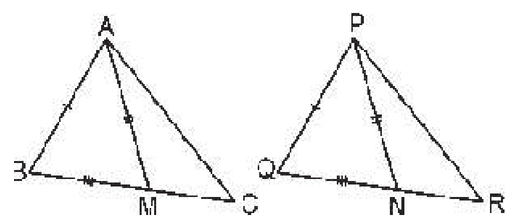
Hence AD bisects ∠A **Proved**

Q.3. *Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of Δ PQR (see Fig.). Show that :*



(i) Δ *ABM* ≅ Δ*PQN* (ii) Δ *ABC* ≅ Δ*PQR*

Sol. (i) In ΔABM and ΔPQN, we have



BM = QN

[∵ BC = QR

$\Rightarrow \frac{1}{2}BC = \frac{1}{2}QR$]

AB = PQ [Given]

AM = PN [Given]

∴ ΔABM ≅ ΔPQN [SSS congruence] **Proved.**

⇒ ∠ABM = ∠PQN [CPCT]

(ii) Now, in ΔABC and ΔPQR, we have

AB = PQ [Given]

∠ABC = ∠PQR [Proved above]

BC = QR [Given]

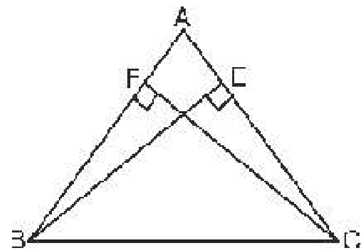
∴ ΔABC ≅ ΔPQR [SAS congruence] **Proved.**

Q.4. *BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.*

Sol. BE and CF are altitudes of a ΔABC.

$\therefore \angle BEC = \angle CFB = 90°$

Now, in right triangles BEB and CFB, we have



Hyp. BC = Hyp. BC [Common]

Side BE= Side CF [Given]

$\therefore$ $\Delta BEC \cong \Delta CFB$ [By RHS congruence rule]

$\therefore \angle BCE = \angle CBF$ [CPCT]

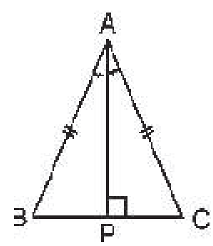
Now, in ΔABC, $\angle B = \angle C$

$\therefore$ AB = AC [Sides opposite to equal angles are equal]

Hence, ΔABC is an isosceles triangle. **Proved.**

Q.5. *ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that $\angle B = \angle C$.*

Sol. Draw AP ⊥ BC.



In ΔABP and ΔACP, we have

AB = AC [Given]

$\angle APB = \angle APC$ [Each = 90°]

AB = AP [Common]

$\therefore$ $\Delta ABP \cong \Delta ACP$ [By RHS congruence rule]

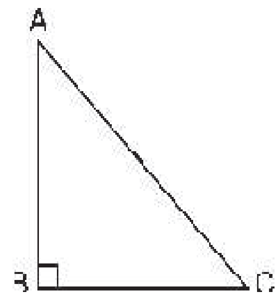
Also, $\angle B = \angle C$ **Proved.** [CPCT]

7 TRIANGLES

EXERCISE 7.4

Q.1. *Show that in a right angled triangle, the hypotenuse is the longest side.*

Sol. ABC is a right triangle, right angled at B.



Now, ∠A + ∠C = 90°

⇒ Angles A and C are each less than 90°.

Now, ∠B > ∠A

⇒ AC > BC ...(i)

[Side opposite to greater angle is longer]

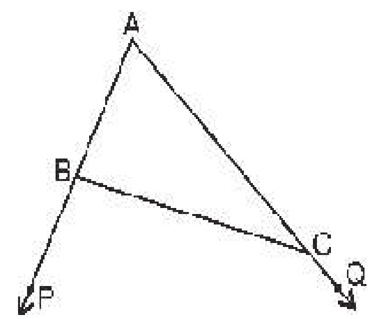
Again, ∠B > ∠C

⇒ AC > AB ...(ii)

[Side opposite to greater angle is longer]

Hence, from (i) and (ii), we can say that AC (Hypotenuse) is the longest side. **Proved**

Q.2. *In the figure, sides AB and AC of Δ ABC are extended to points P and Q respectively. Also, ∠ PBC < ∠QCB. Show that AC > AB.*

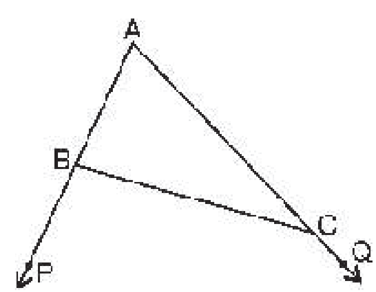


Sol. ∠ABC + ∠PBC = 180° [Linear pair]

⇒ ∠ABC = 180° – ∠PBC ...(i)

Similarly, ∠ACB = 180° – ∠QCB ...(ii)

It is given that ∠PBC < ∠QCB



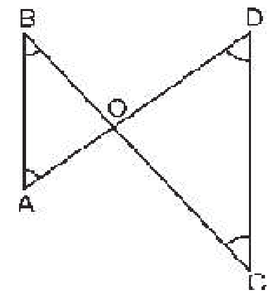
∴ 180° – ∠QCB < 180° – ∠PBC

Or ∠ACB < ∠ABC [From (i) and (ii)]

⇒ AB < AC

⇒ AC > AB **Proved.**

Q.3. *In the figure, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.*



Sol. $\angle B < \angle A$ [Given]

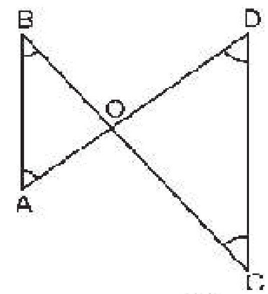
$BO > AO$ ...(i)

[Side opposite to greater angle is longer]

$\angle C < \angle D$ [Given]

$\Rightarrow CO > DO$ ...(ii)

[Same reason]



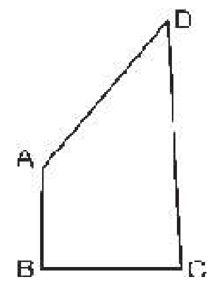
Adding (i) and (ii)

$BO + CO > AO + DO$

$\Rightarrow BC > AD$

$AD < BC$ **Proved.**

Q.4. *AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see Fig.). Show that $\angle A > \angle C$ and $\angle B > \angle D$.*



Sol. Join AC.

Mark the angles as shown in the figure.

In ΔABC,

$BC > AB$ [AB is the shortest side]

$\Rightarrow \angle 2 > \angle 4$ ...(i)

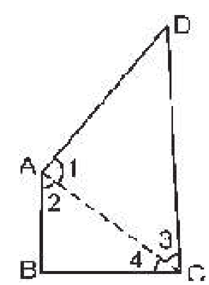
[Angle opposite to longer side is greater]

In ΔADC,

$CD > AD$ [CD is the longest side]

$\angle 1 > \angle 3$ ...(ii)

[Angle opposite to longer side is greater]



Adding (i) and (ii), we have

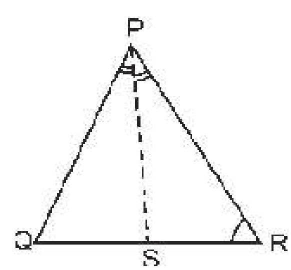
$\angle 2 + \angle 1 > \angle 4 + \angle 3$

$\Rightarrow \angle A > \angle C$ **Proved.**

Similarly, by joining BD, we can prove that

$\angle B > \angle D$.

Q.5. *In the figure, $PR > PQ$ and PS bisects $\angle QPR$. Prove that $\angle PSR > \angle PSQ$.*

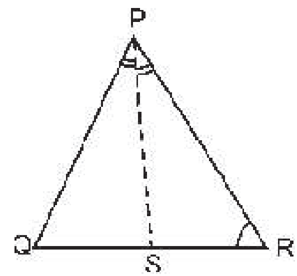


Sol. $PR > PQ$

$\angle PQR > \angle PRQ$ ...(i)

[Angle opposite to longer side is greater]

$\angle QPS > \angle RPS$ [$\because$ PS bisects $\angle QPR$] ...(ii)



In ΔPQS, $\angle PQS + \angle QPS + \angle PSQ = 180°$

$\Rightarrow \angle PSQ = 180° - (\angle PQS + \angle QPS)$ ...(iii)

Similarly in ΔPRS, $\Delta PSR = 180° - (\angle PRS + \angle RPS)$

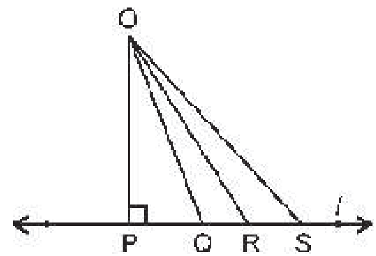
$\Rightarrow \angle PSR = 180° - (\angle PRS + \angle QPS)$ [from (ii) ... (iv)

From (i), we know that $\angle PQS < \angle PSR$

So from (iii) and Iiv), $\angle PSQ < \angle PSR$

$\Rightarrow \angle PSR > \angle PSQ$ **Proved**

Q.6. *Show that of all the segments drawn from a given point not on it, the perpendicular line segment is the shortest.*



Sol. We have A line $\overleftrightarrow{l}$ and O is a point not on $\overleftrightarrow{l}$.

$OP \perp \overleftrightarrow{l}$.

We have to prove that $OP < OQ$, $OP < OR$ and $OP < OS$.

In ΔOPQ, $\angle P = 90°$

$\therefore$ $\angle Q$ is an acute angle (i.e., $\angle Q < 90°$]

$\therefore$ $\angle Q < \angle P$

Hence, $OP < OQ$ [Side opposite to greater angle is longer]

Similarly, we can prove that OP is shorter than OR, OS etc. **Proved.**

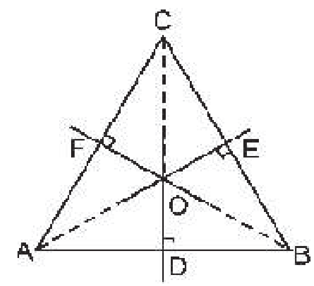
7 TRIANGLES

EXERCISE 7.5 (OPTIONAL)

Q.1. *ABC is a triangle. Locate a point in the interior of ΔABC which is equidistant from all the vertices of ΔABC.*

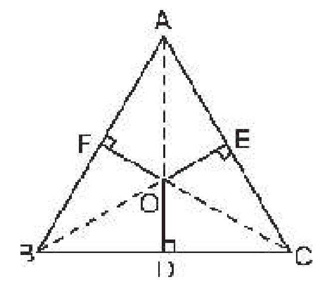
Sol. Draw perpendicular bisectors of sides AB, BC and CA, which meets at O.

Hence, O is the required point.



Q.2. *In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.*

Sol.



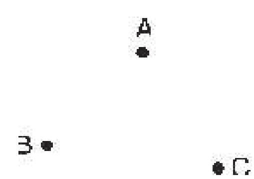
Q.3. *In a huge park, people are concentrated at three points (see Fig.).*

A : where there are different slides and swings for children,

B : near which a man-made lake is situated,

C : which is near to a large parking and exit.

Where should an icecream parlour be set up so that maximum number of persons can approach it?

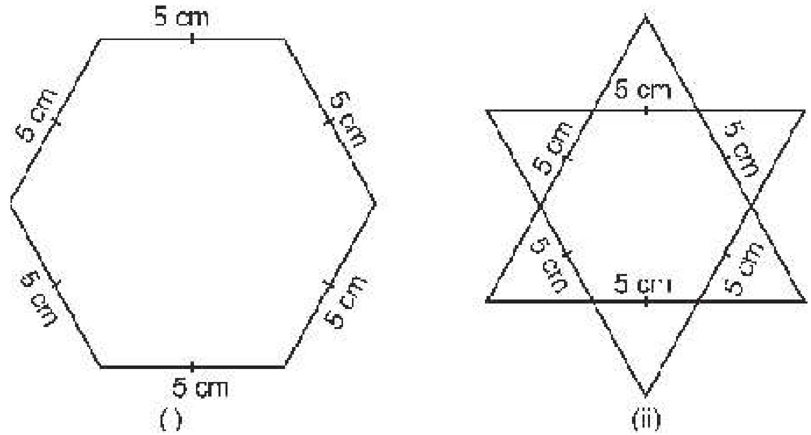


Draw bisectors $\angle A$, $\angle B$ and $\angle C$ of ΔABC. Let these angle bisectors meet at O.

O is the required point.

Sol. Join AB, BC and CA to get a triangle ABC. Draw the perpendicular bisector of AB and BC. Let they meet at O. Then O is equidistant from A, B and C. Hence, the icecream pra

Q.4. *Complete the hexagonal and star shaped Rangolies [see Fig. (i) and (ii)] by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?*



Sol.